Revista de Ştiinţe Politice. Revue des Sciences Politiques • No. 73 • 2022: 229 - 240

ORIGINAL PAPER

Efficient frontier in portfolios containing stock market and financial digital assets

Andrei-Dragoş Popescu[1)]

Abstract:
The traditional finance infrastructure has been adapting and evolving with great speed in the last decade, due to the advancement of technologies and the performance of alternative financial models.
The paradigm shift that was presented due to the innovative models of financial digital assets, within the spectrum of Decentralized Finance (DeFi), has pushed investment portfolio structures to a new level of complexity, as they integrated the space of alternative investments and took a deep dive into an unregulated environment that presents new opportunities and challenges.
The risk associated to these new assets is directly proportional with the returns and opportunities that arise from such complex structures.
Fund managers and financial analysists are always embracing novel technologies that optimize, enhance and improve the way financial instruments perform and hedge for the best outcome. The space of financial digital assets introduced new variables for liquidity and volatility along with the connection to new possibilities of diversification in a decentralized environment.
This paper will analyze an innovative approach of a portfolio construction that is composed of stock market and financial digital assets. We will be assessing the Return on Investment (ROI) in the context of combining high volatility crypto assets with a traditional stocks portfolio, in order to identify the best opportunities that present themselves in this hybrid scenario.
The aim of this paper is to explore the balance between returns and risk management by means of an efficient frontier model. We will provide an in-depth analysis for the best performing portfolio taking in consideration volatility and market capitalization of the assets used in this study.

Key words: Financial Digital Assets, Stock Market Assets, Efficient Frontier.
JEL Classification: F33, F42, F01, E02.

[1)] PhD Student, University of Craiova, Department of Finance, Craiova, Romania, Phone: +6585886565, Email: andrei@scx.holdings. https://orcid.org/0000-0002-9048-3055.

Introduction

In the last years, digital assets trading has been consolidating, reaching greater volumes and liquidity, as more users are understanding and adopting the underlying technology that drives the crypto assets.

The dynamic evolution and adoption of these emerging markets presented high volatility, which was translated in high risk assessment for the participants, and a consequence for these factors conferred higher returns compared to the traditional markets. As the markets matured and more seasoned investors joined, the symmetry of exposures to the risks associated to this environment decreased.

In our opinion we have applied the right methodology, in order to identify the best way to optimize a balanced portfolio, by calculating the efficient frontier for both of these spaces.

The efficient frontier is a method that originates from the modern portfolio theory, that focuses on achieving the balance between returns and risks, for a set of assets seeking to outline the highest performance. This method was formulated by Harry Markowitz in 1952, and was elaborated in a later study by the same author in 1959.

The theory behind our research is based on the assumption that given all the different assets in an investment universe, there is an optimal portfolio for any investor depending on their level of risk, and that portfolio consists of some proportion of stocks and financial digital assets.

This method optimizes the portfolio weights for a set of assets in an investment universe, minimizing the risk for each return assumed.

The essence of the modern portfolio theory is that it is possible to construct an efficient frontier of optimal portfolios, that offer the maximum expected return for a given level of risk. The popular notion which derived from this theory is that, the higher the risk, the higher the return.

Literature review

The literature strand with regards to the Efficient Frontier model present many setbacks or limitations on the theory. One of the limitations of this theory considers that it is best to be considered a theory and not to be applied in practical applications. The efficient frontier theory is based on an assumption model and because of this, it cannot be fully utilized with accurate results in certain applications.

One limitation of the theory is based on the fact that the investments making up the optimal portfolio in the efficient frontier are selected based on the assumption that returns on assets usually follow normal distribution. Another assumption is that investors always consider their allocations rationally, minimizing risk, and furthermore, the asset prices behave in the same manner, meaning they follow a normal distributed way.

In reality, returns on assets are said to follow a heavy-tailed distribution and not the normal distribution.

Aside from the above limitations, some underlying assumptions in the efficient theory such as the rationality of investors and their tendency to avoid risk is debatable. This is because the market generally witness irrational investors and risk-daring investors who can influence certain decisions in the market.

Clara Calvo, Carlos Ivorra, and Vicente Liern (2012) studied the efficient frontier of the portfolio selection problem and their research concluded that the shape of the efficient frontier can present many irregularities which must be taken into account since the risk of an efficient portfolio can be very sensitive to the selected expected return. This scenario applies especially for small-sized problems. Their study reveals that as the number of assets increases, the efficient frontier becomes more regular at a large scale, and hence its "microscopic" irregularities are not relevant.

Quintana and Moreno (2021) analyzed how the efficient frontier model is used for different scenarios, using multi-objective evolutionary algorithms. Their research is focused on the differences in the number of portfolios or their spacing along the Pareto front, and they introduce a set of alternatives in resampling with standard multi-objective evolutionary algorithms under real-world constraints.

Clarence C.Y. Kwan (2003) researched familiar portfolio concepts that enabled investors to achieve higher expected returns without additional risk exposure, by pooling of investment capital. Based on his study, the risk of the pooled investment must be a particular weighted average of the participating investors' preferred risks.

Pedersen et al. (2020) approached the efficient frontier analysis from an Environmental, Social and Governance (ESG) model. In their study the solution to the investor's portfolio problem is defined by an ESG-efficient frontier and their results show the costs and benefits of responsible investing, analyzing the highest attainable Sharpe Ratio for each ESG level.

The efficient frontier theory was researched and calculated using deep neural networks in some Mean-Variance and Mean-CVaR portfolio optimization problems by Warin X. (2021). In his study, Warin concluded that by adding additional constraints, in order to compare different formulations, he presented the results from the perspective of a new projected feedforward network that is able to deal with some global constraints on the weights of the portfolio while outperforming classical penalization methods.

Different formulas, techniques and parameters have been used to optimize the modeling of the efficient frontier. For instance, Bauder et al. (2018) analyzed the expected return, the variance of the global minimum variance portfolio, along with the slope parameter, as the main parameters for his study of the efficient frontier from a Bayesian perspective. Their posterior distribution is derived by assigning the diffuse and the conjugate priors to the mean vector and the covariance matrix of the asset returns and is presented in terms of a stochastic representation. Furthermore, Bayesian estimates together with the standard uncertainties for all three parameters are provided, and their asymptotic distributions are established.

Methodology

The efficient frontier is one of the earliest portfolio management models used for optimizing portfolios.

In this model, we have to decide which are the optimal weights for a set of stocks in concordance with the behavior over time of the portfolio.

Our first step in our study is to define the weights of our portfolio.

Let N be the number of assets selected for our investment, and based on these, the weights are defined as follows:

$$\sum_{i=1}^{N} w_i = 1$$

Equation 1 - Weight of the set of assets in portfolio

In equation 1, w_i represents the weight for a stock asset in our portfolio, and all weights must sum 1, given that we define the percentage of our trading capital to be settled on different asset *i*.

We will also define the market return of the proposed asset and the this could be defined as the percent change of the asset:

$$return = \frac{p_{i-1} - p_i}{p_i}$$

Equation 2 - Market return by percent change

We have defined the two basic concepts to develop our study and this allows us to construct our portfolio.

In equation 3 we display the formula that allows us to calculate the return of the portfolio:

$$mean\ returns = \frac{\Sigma returns}{N_{days}}$$

Equation 3 - Average of returns for an asset

First, we must calculate the mean return of each stock, which is a vector of *N x 1* with the mean return of each stock, and this will allow us to calculate the expected return of the portfolio with the given weights.

$$Expected\ Return = \sum_{i=1}^{N} mean\ returns * weights * trading\ days$$

Equation 4 - Expected Return for an asset

We will note that all these operations are matrix products, and the next necessary step that we need to determine is the expected volatility:

$$Expected\ Volatiliy = \sqrt{w^T \cdot cov(returns) * trading\ days \cdot w}$$

Equation 5 - Expected Volatility for an asset

As we have all the necessary metrics, we advance in computing the sharpe ratio, which is a proportion of return/risk in our stock, and this will help us calculate our efficient frontier for a given number of random portfolios:

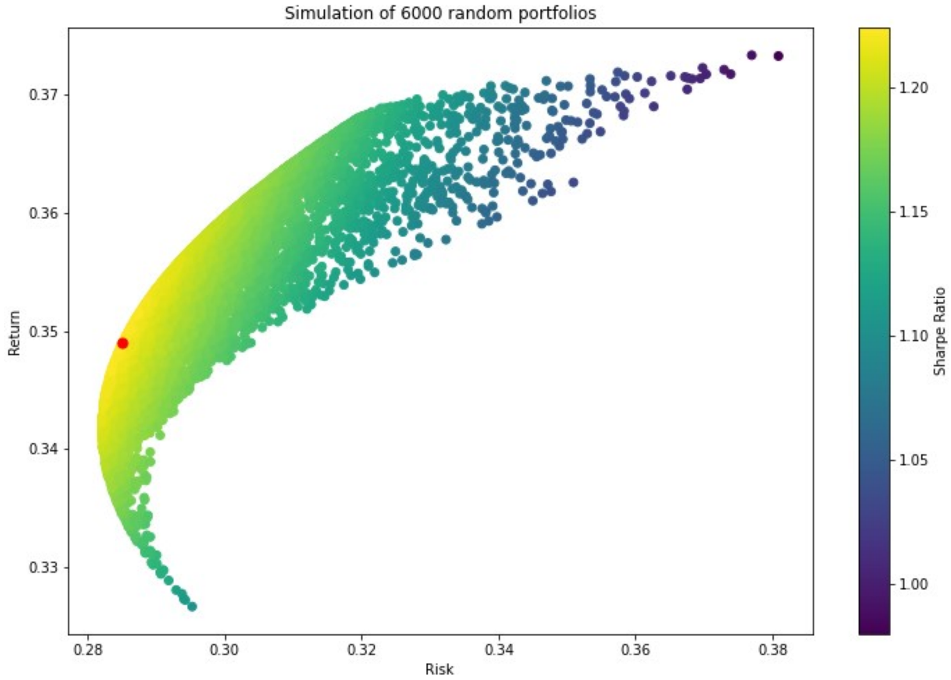
$$Sharpe\ Ratio = \frac{Expected\ Return}{Expected\ Volatility}$$

Equation 6 - Sharpe ratio for an asset

Based on the above formulas, we can simulate several portfolios from which we can extract the maximized sharpe ratio and find the ideal weights for a given return and volatility.

As a continuation of our study, we can generate random portfolios with random weights, and we can present the efficient frontier in a graphical illustration. In the figures generated bellow, the red dot represents the optimal point.

Figure 1 – Simulation of 6000 random portfolios



Source: Author's creation

We can also find an optimal sharpe ratio for our portfolio changing the weights for every assumed return.

The efficient frontier can be modeled as an optimization problem with the next conditions:

$$SR_{max} = -min\left(-SR(w_i)\right)$$

Equation 7 - Optimization Problem

Equation 8 is defining the constraints:

$$0 \leq w_i \leq 1; \sum_{i=1}^{N} w_i = 1$$

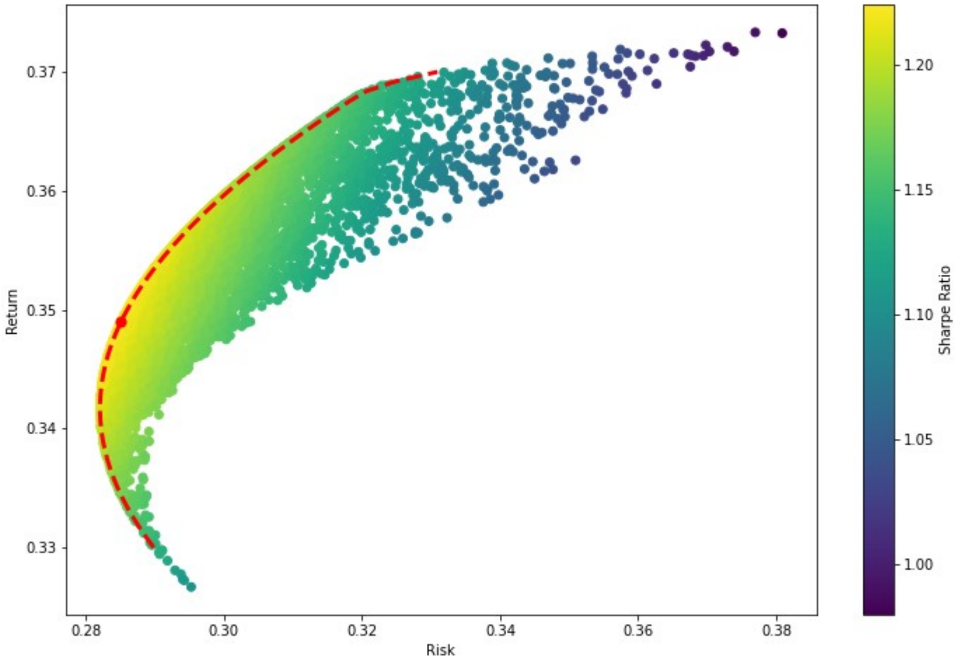
Equation 8 - Constraints for optimization problem

As we cannot control the market returns, nor the volatility, the only control variables we can establish are the weights of each stock in our portfolio.

Now we can use different methods to solve this optimization problem and we decided to use Sequential Least Squares which is implemented in scipy library for python language.

Our study is aiming to calculate the optimal weights to get the less risk for a given expected return, and this is what we interpret as the efficient frontier.
Figure 2 illustrates the efficient frontier.

Figure 2 – Efficient Frontier



Source: Author´s Creation

The efficient frontier solution allows us to identify those portfolios that are assuming more risk than they should for an expected market return, therefore, these portfolios are not efficient and only those that are on the efficient frontier fulfil the requirements.

To test the case study, we will form three investment portfolios: the first with three stock market assets, the second with three digital assets and the last will be a combination of the six previous assets.

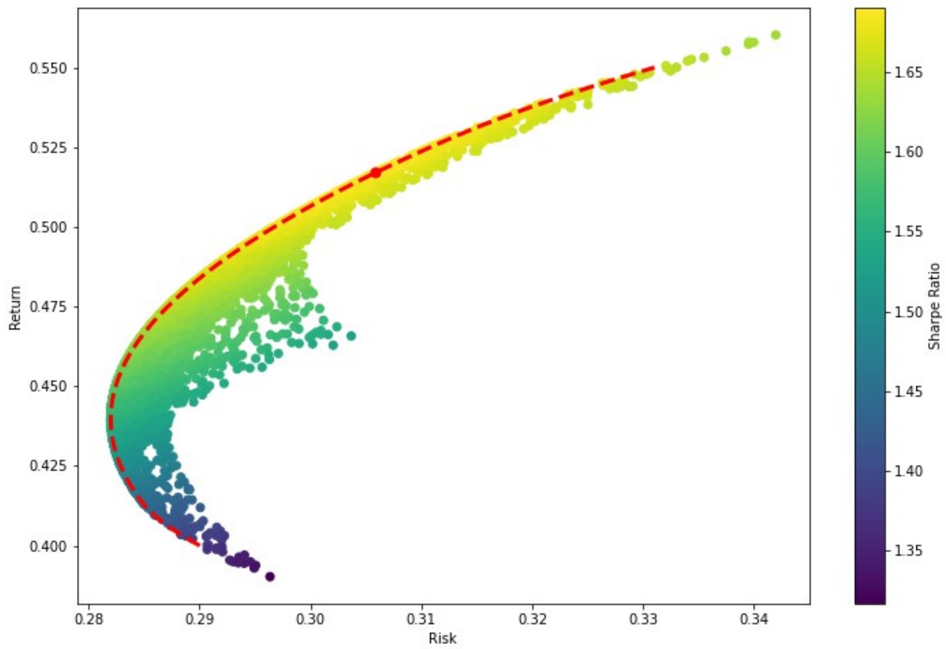
For the benefit of this study, we have identified and used three stocks: Apple (AAPL), Microsoft (MSFT) and Google (GOOG).
For the first portfolio, we select the next stocks:

Table 1 – Stock Market Assets with Market Capitalization

ASSET	MARKET CAPITALIZATION
APPLE (AAPL)	2.94 T USD
MICROSOFT (MSFT)	2.57 T USD
GOOGLE (GOOG)	1.97 T USD

Source: Author's creation with data taken from Yahoo Finance

Figure 3 – Efficient Frontier for portfolio 1



Source: Author´s Creation

Once the efficient frontier and the optimal point have been calculated for this portfolio, the maximized value for the **Sharpe Ratio is 1.69**, on the table 2 the weights for each asset are reported.

Table 2 – Stock Market Assets with their weights

ASSET	WEIGHT IN PORTFOLIO
APPLE (AAPL)	0.59
MICROSOFT (MSFT)	0.35
GOOGLE (GOOG)	0.06

Source: Author's creation

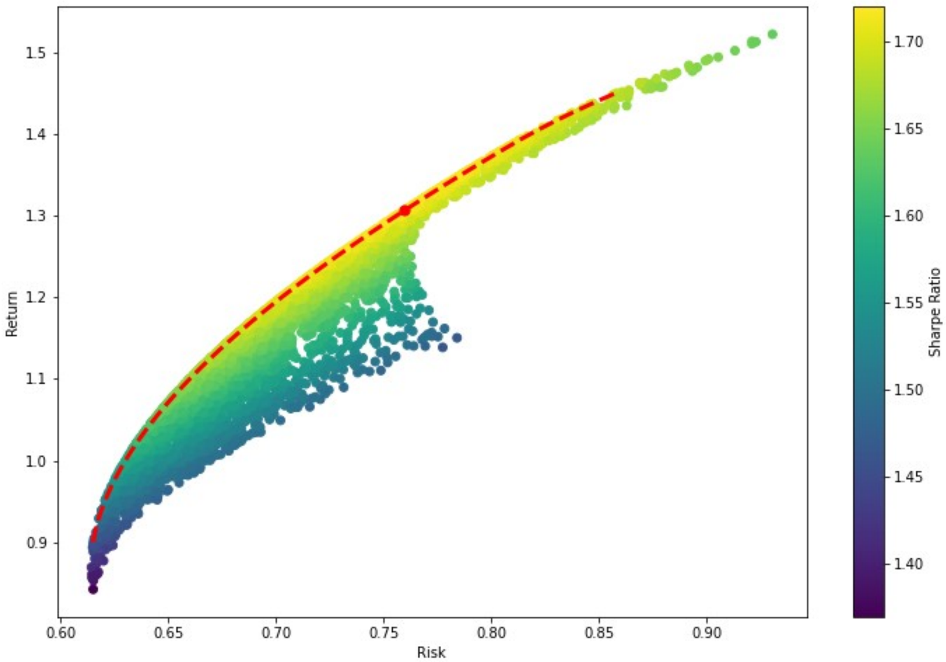
The second portfolio we have selected, includes the next digital assets:

Table 3 – Financial Digital Assets with Market Capitalization

ASSET	MARKET CAPITALIZATION
BITCOIN (BTC)	932 B USDT
ETHEREUM (ETH)	482 B USDT
BINANCE COIN (BNB)	94 B USDT

Source: Author's creation with data taken from Coinmarketcap.com

Figure 4 – Efficient frontier for portfolio 2



Source: Author´s Creation

Making the relevant calculations, the result for maximized **Sharpe Ratio is 1.72**, the weights for each asset on this portfolio are reported on table 4.

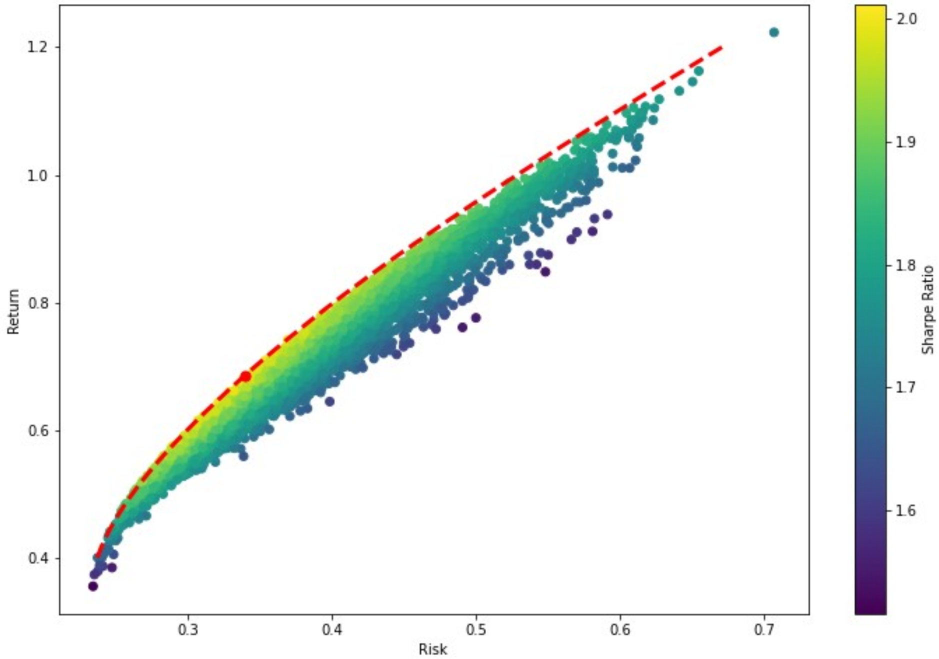
Table 4 – Financial Digital Assets with their weights

ASSET	WEIGHT IN PORTFOLIO
BITCOIN (BTC)	0.17
ETHEREUM (ETH)	0.29
BINANCE COIN (BNB)	0.54

Source: Author's creation

The third portfolio is a combination of the 6 previous assets.

Figure 5 – Efficient frontier for portfolio 3



Source: Author's creation

Finally, for the combined portfolio, the value for **Sharpe Ratio is 2.0**, and the weights for each asset are reported on table 5.

Table 5 – Mixed portfolio of Stock and Financial Digital Assets with their weights

ASSET	WEIGHT IN PORTFOLIO
APPLE (AAPL)	0.4
MICROSOFT (MSFT)	0.23
GOOGLE (GOOG)	0.02
BITCOIN (BTC)	0.08
ETHEREUM (ETH)	0.06
BINANCE COIN (BNB)	0.21

Source: Author's creation

Results

The stock market is characterized by having a low volatility and sustained trends over time, this causes the risk to decrease, in the other hand, digital assets usually have greater volatility, but at the same time a higher return and performance.

A low sharpe ratio is interpreted as a high-risk investment, since proportionally, the expected return is much lower than the risk. However, a high sharpe ratio shows that it is a low-risk investment, as the expected return is greater than the risk.

Table 6 – Portfolio and sharpe ratio

Portfolio	Sharpe Ratio
Portfolio 1 – Stock Market Assets	1.69
Portfolio 2 – Digital Assets	1.72
Portfolio 3 – Mixed Assets	2.0

Source: Author's creation

The stock market and digital assets behave differently over time as each one is characterized by having a different volatility and returns. For this reason, the portfolio of stocks and digital assets presented a different sharpe ratio, and we can observe a higher sharpe ratio when we combine the assets. The result clearly shows the best-case scenario, as we optimize the best option of these two environments and assets.

Conclusion

The efficient frontier is an approach to model the behavior of a portfolio after selecting certain investment assets, maximizing sharpe ratio.

The model makes assumptions that all investors make logical decisions minimizing risk and that the price follows a normal distribution over time.

The theory behind the Efficient Frontier and Optimal Portfolios states that we can identify the most favorable combination of risk and return.

The theory relies on the assumption that investors prefer portfolios that generate the most substantial possible return with the least amount of involved risk. We refer to these type of scenarios as optimal portfolios, and they form the efficient frontier curve.

Our study is based on the fact that different combinations of assets produce different levels of return. The optimal portfolio concept represents the best of these combinations, those that provide the maximum possible expected return for a given level of acceptable risk.

The scenario behind our study is based on a buy and hold strategy, so one important question that needs to be addressed is the correlation factor between the assets.

The relationship between assets is an essential part of the optimal portfolio theory. Some prices move in the same direction under similar circumstances, while others go in opposite directions. The more out of sync these price developments are, the lower the covariance between two assets is, which translates into lower overall risk.

The main reason why diversifying is important, comes down to minimization of the volatility and overall exposure to a single asset. This relies specifically to the financial digital assets market, where we encounter different types of risks. On this note, we can address two different type of risks, which are associated with any crypto asset: the first one is the market risk, where within the crypto ecosystem most of the crypto assets ebb and flow with the market as a whole, having bitcoin as the main market mover; and the second one is the idiosyncratic risk, which is crypto specific (Popescu, 2020b). The idiosyncratic risk reflects directly to DeFi, as it is vulnerable to fraud as well as to the proliferation of untested financial innovations (Popescu, 2020a).

Looking at the performance of the stock market from 2001 to 2010, the buy and hold strategy was hardly effective and this led to many people claiming that they could not rely on the efficient frontier anymore. The reason for these opinions came from the fact that the market moved in a range period. The efficient frontier proved valid as the performance of the stock market from 2010 to date, has been phenomenal, with an impressive growth.

With regards to the financial digital assets, specifically crypto assets, we believe that this asset class has proven itself to be an exceptional selection for alternative allocations. Financial digital assets present themselves as high volatility assets, with high returns, which are not correlated with the momentum and moves of traditional assets. As the market matures, some investors look at the crypto assets as a hedge for the current traditional portfolios and they are slowly being considered as a good allocation. The mindset and adoption for this new asset class will grow over time, as institutional investors keep researching the space and regulations for this environment are being formulated. There are still a lot of uncertainties with regards to the regulatory framework, as the technology that drives these new assets is being acknowledged. Once these hurdles will be passed, we believe that these novel digital assets will become a norm in most of the portfolio constructions.

References:

Bauder, D., Bodnar, R., Bodnar, T., Schmid, W., (2018). Bayesian estimation of the efficient frontier. *Scandinavian Journal of Statistics*, Theory and Applications, Vol 43, Issue 3. https://doi.org/10.1111/sjos.12372

Calvo, C., Ivorra, C., Liern, V. (2012). On the Computation of the Efficient Frontier of the Portfolio Selection Problem. *Journal of Applied Mathematics*, vol. 2012, 25 pages, 2012. https://doi.org/10.1155/2012/105616

Clarence, C.Y. Kwan, (2003). Improving the Efficient Frontier, *The Journal of Portfolio Management,* Jan 2003, 29 (2) 69-79; DOI: 10.3905/jpm.2003.319874

Coinmarketcap, (2021). *Market Capitalization*, Data from 12th December 2021. Retrieved from https://coinmarketcap.com/

H. M., Markowitz, (1952). Portfolio Selection, *Journal of Finance*, vol. 7, pp. 79–91, 1952.

H. M., Markowitz, (1959). *Portfolio Selection: Efficient Diversification of Investments*, vol. 16, Cowles Foundation for Research in Economics at Yale University, John Wiley & Sons, New York, NY, USA.

Pedersen, L. H., Fitzgibbons, S., Pomorski, L. (2020). Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics*, Volume 142, Issue 2, November 2021, Pages 572-597. https://doi.org/10.1016/j.jfineco.2020.11.001

Popescu, AD. (2020a). Decentralized Finance (DeFi) – The Lego of Finance, *Social Sciences and Education Research Review*, Volume 7, Issue 1, pg.321-349, https://sserr.ro/wp-content/uploads/2020/07/SSERR_2020_7_1_321_349.pdf, ISSN 2393–1264, ISSN–L 2392–9863

Popescu, AD. (2020b). Transitions and concepts within Decentralized Finance (Defi) Space, Research Terminals in the Social Sciences, SITECH Publishing House, *The Proceedings of CIL 2020: Ninth Edition of International Conference of Humanities and Social Sciences - Creativity, Imaginary*, Language, pg.40-61, https://www.cilconference.ro/wp-content/uploads/2020/09/cil_2020_research_terminals.pdf#page=40, ISBN 978-606-11-7565-9.

Popescu, AD. (2021). Non-Fungible Tokens (NFT) – Innovation beyond the craze, Revues et Méthodes sur le Management et le Commerce International, *Proceedings of Engineering & Technology – PET*, Volume 66, pg.26-30, http://ipco-co.com/PET_Journal/IBEM-Proceedings/IBEM%20PROCEEDINGS.pdf#page=31, ISSN 1737-9334

Quintana, D., Moreno, D. (2021). *Resampled Efficient Frontier Integration for MOEAs*. Entropy, 23, 422. https://doi.org/10.3390/e23040422

Warin, X. (2021). *Deep learning for efficient frontier calculation in finance*, arXiv preprint arXiv:2101.02044. https://arxiv.org/pdf/2101.02044v2.pdf

Yahoo Finance, (2021). *Market Capitalization*, Data from 12th December 2021. Retrieved from https://sg.finance.yahoo.com/

Article Info

Received: March 01 2022
Accepted: March 09 2022

How to cite this article:
Popescu, A.D. (2022). Efficient frontier in portfolios containing stock market and financial digital assets. *Revista de Ştiinţe Politice. Revue des Sciences Politiques*, no. 73/2022, 229-240.